Priority Queues and Heaps

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Array of Talks

- Title: "JUST THE HACKERS YOU NEED"
- Location: HFH 1132 (CS conference room)
- Time: 3:30 (after class)

Outline

- Monday Recap
- More Tree Properties
- Priority Queue
- Heaps

MONDAY RECAP

$O(n^2)$ Sorting Algorithms

- Bubble sort
  - Bubble the largest element to the end in each pass
- Insertion sort
  - Insert the next element into the sorted portion of the list
- Selection sort
  - Find the smallest item and put it in its proper location

O(nlog(n)) Sort Algorithms

- Merge Sort
  - Break the problem up until you have 1 or 2 items and put them in order
  - Merge the sorted lists O(k) where k is the size of the small lists
  - T(n) = 2T(n/2) + O(n) === O(n*log(n)) (masters theorem)

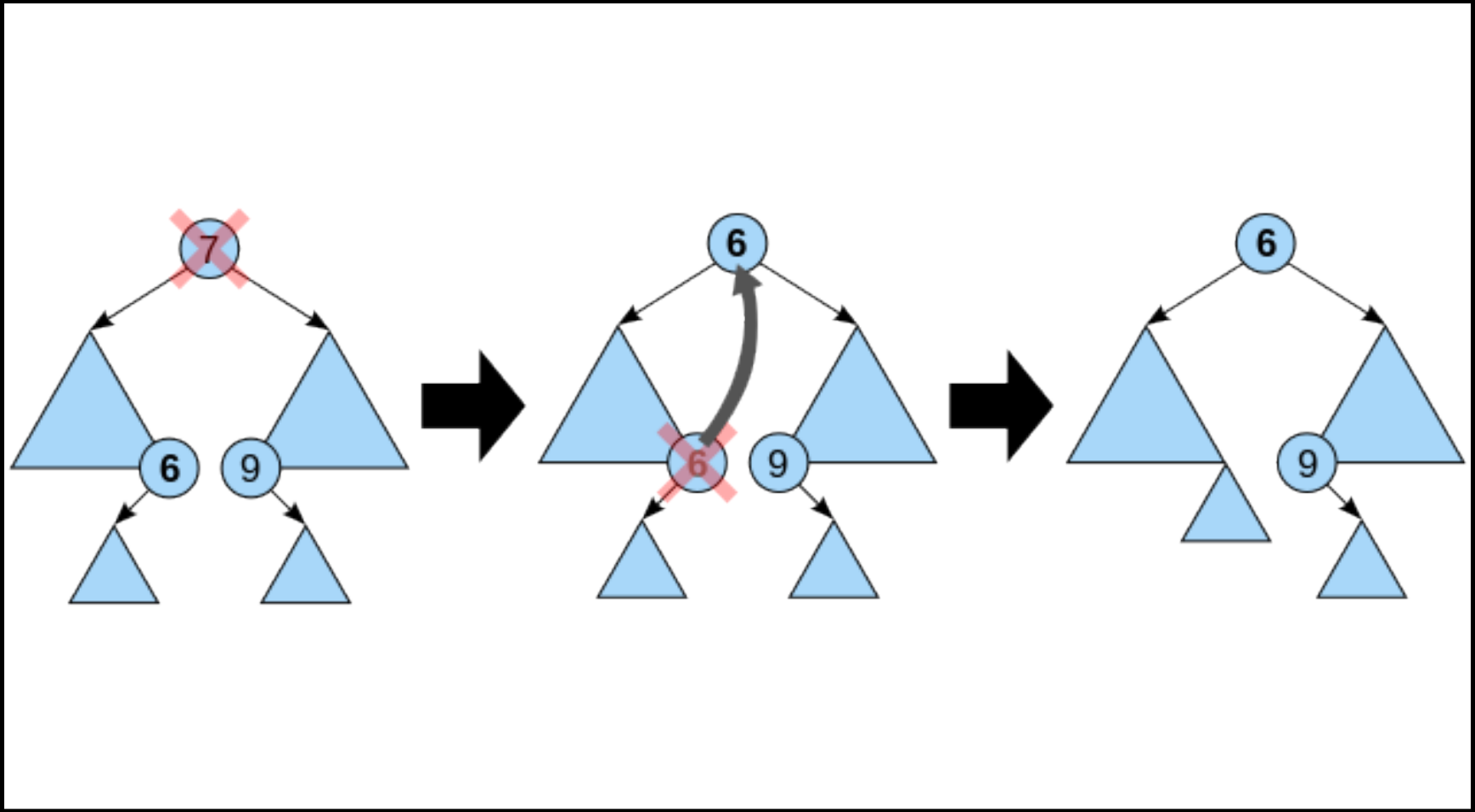
BST Remove

- If the node has no children simply remove it
  - Parent should point to NULL
- If the node has a single child, update its parent to point to its child and remove the node
  - Recursive helper function should return the pointer to the new node (in addition to the removed value)

Removing a node with two children

- Replace the value of the node with the largest value in its left-subtree (right-most descendant on the left hand side)
  - Make sure to save a copy of the original node to return
- Then repeat the remove procedure to remove the node whose value was used in the replacement
  - Ignore the value of the removed node as it is still in the tree

Removing a node with two children

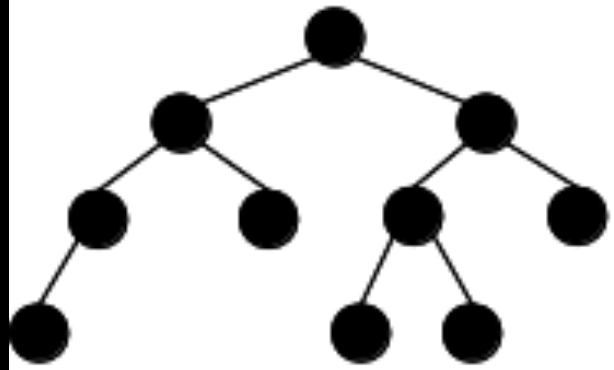


MORE TREE PROPERTIES

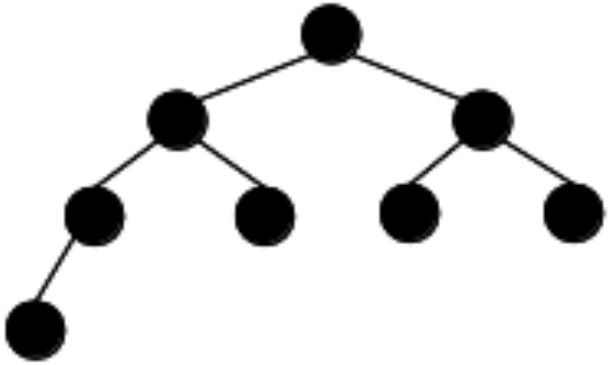
Full Binary Tree

- Full Binary Tree
  - A binary tree in which every node has either 0 or 2 children
- Complete Binary Tree
  - A binary tree in which every level is completely filled save for the last where all items must be filled starting with the left-hand-side
    - Complete trees are always balanced

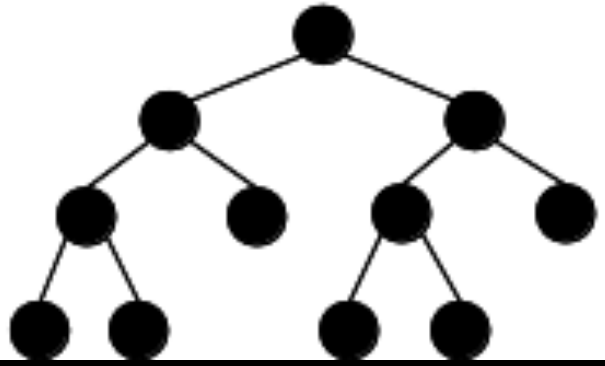
Neither complete nor full



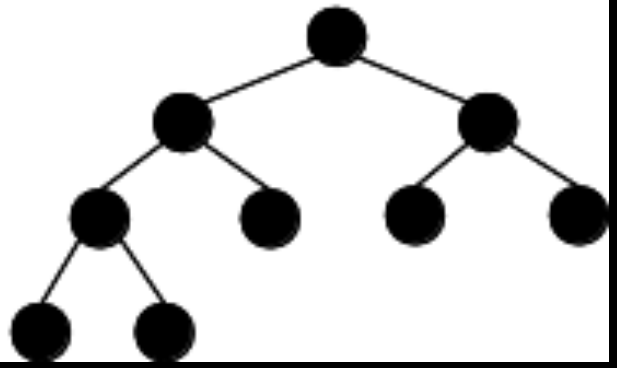
Complete but not full



Full but not complete



Complete and full



PRIORITY QUEUE

Priority Queue

- Abstract data type with operations similar to the queue
  - enqueue(item, **priority**)
    - Add an item to the queue
  - dequeue()
    - Remove the item with highest priority from the queue (undefined order for items with same priority)

Brainstorm

- Discuss for a few minutes with those near you:
  - Attempt to come up with two functionally distinct ways to implement a priority queue as a modification, or combination of the simple structures we've learned in this class

Bounded Priority Queue

- Simple trick if the number of priorities is bounded
- Provide one FIFO queue for each priority
- Always look for items starting from the highest queue before proceeding to the next
- **enqueue**: O(1)
- **dequeue:** O(1)

Sorted List Implementation

- Use a standard linked-list implementation of a queue
- Modify enqueue such that it places the item in the appropriate location in the list according to its priority
- **enqueue**: O(n)
- **dequeue:** O(1)

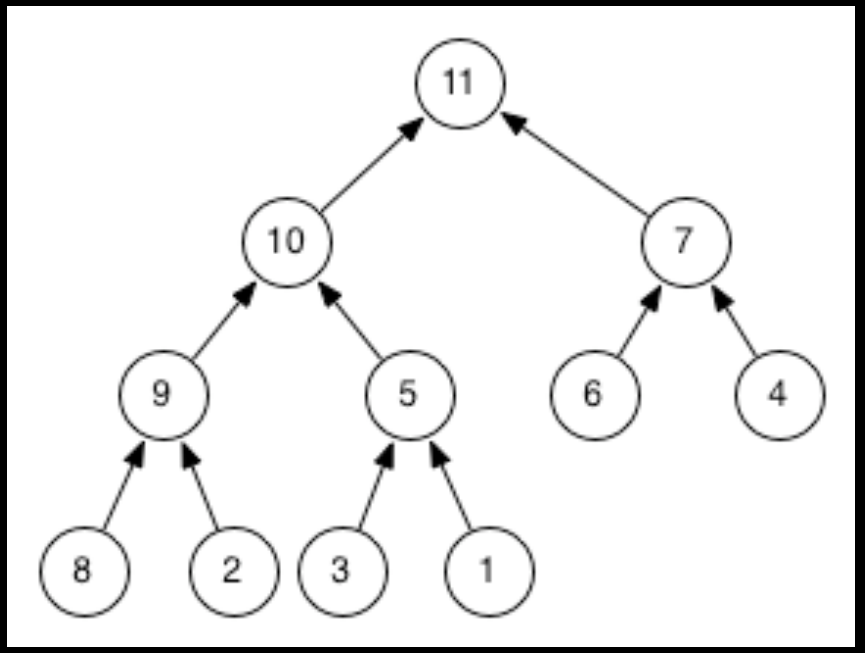
Tree Implementation

- Construct a tree such that its root is always the highest priority
- Additionally every subtree has the same property (parent is of equal or higher priority than its children)

HEAP

Heap

- A **complete** tree where each node's parent has a higher or equal priority



Think about it

- Are there any trees that can be both a heap and a BST?

Heap Insertion (enqueue)

- Initially insert item in the next free location (obey the completeness property)
- Continue to swap it with its parents until the ordering property is valid (bubble up)
- **enqueue**: O(log(n)) – worst cases traverses up the entire height of the tree

Heap removal (dequeue)

- Store the value at the root to return at the end
- Swap the last item in the tree with the root
- Continually swap the current node with its child of highest priority until it is of higher priority than both children (bubble-down)
- **dequeue**: O(log(n)) – worst cases traverses down the entire height of the tree

Lab 9

- Write function to test if array is in heap-order
- Arrays are great for storing complete trees

| 11 | 10 | 7 | 9 | 5 | 6 | 4 | 8 | 2 | 3 | 1 |